

Course title: *MA493 – Introduction to Hilbert Spaces*

Instructor: Patrick L. Combettes, plc@math.ncsu.edu

Term: Fall 2024

Time: Tuesdays and Thursdays, from 15:00 to 16:15

Office: SAS 3276

Office hours: Tuesdays and Thursdays, from 16:30 to 17:30 (or anytime by appointment)

Prerequisite: MA242, MA305 or equivalent

Course objectives: This course provides an introduction to the basic theory of Hilbert spaces and to some of its applications.

Course description: Hilbert space theory constitutes the core around which most of modern analysis has grown, with a wealth of applications in areas such as variational analysis, differential equations, harmonic analysis, control, inverse problems, and data analysis. A Hilbert space is a powerful extension of the basic notion of a Euclidean plane, from which it retains many intuitive concepts such as orthogonality, parallelogram and polarization identities, orthogonal coordinate decompositions, projections and best approximation concepts, as well as linear vector transformations.

Content: Vector spaces, metric spaces, normed spaces, Hilbert spaces, projection theorem, orthonormal systems, orthonormal bases, Fourier series, linear operators on Hilbert spaces. Applications to differential equations, signal theory, optimization, physics, machine learning, inverse problems, and finance.

Grading: Homework 30%, midterm exams 30%, final exam 40%.

Reference material (no purchase necessary):

- L. Debnath and P. Mikusinski, *Introduction to Hilbert Spaces with Applications*, 3rd. ed. Elsevier, 2000.
- N. Young, *An Introduction to Hilbert Space*. Springer, 1988.