Course title: *MA515 – Functional Analysis*

Instructor: Patrick L. Combettes, plc@math.ncsu.edu

Term: Spring 2027.

Time: Tuesdays and Thursdays, 11:45–13:00, SAS1220

Office : SAS 3276.

Office hours: By appointment.

Course objectives: Functional analysis is the branch of mathematics that supports the study and the numerical solution solve both linear and nonlinear problems posed on a normed space that is no longer finite-dimensional, a situation that arises in many concrete problems and areas such as approximation theory, optimization, the calculus of variations, partial differential equations, game theory, inverse problems, machine learning, statistics, and control. Topics include normed spaces, completeness, functionals, the Hahn-Banach Theorem, duality, linear operators operators, and Hilbert spaces.

Prerequisite: MA426 or equivalent.

Content:

- Introduction: General notation, set theory, sequences on the real line.
- Metric spaces: definition and examples, topology, compactness, continuity, completeness, the Banach-Picard theorem, the Baire category theorem, Arzelà–Ascoli theorem.
- Normed spaces: Vector spaces, Banach spaces, linear operators, duality, weak convergence of sequences, Hahn–Banach theorem, open mapping theorem, closed graph theorem, Banach-Steinhaus theorem.
- Hilbert spaces: Pre-Hilbert spaces, basic properties, orthogonality, projections, orthonormal sets, Hilbert spaces, projections, Hilbertian bases, weak convergence.
- Linear operators on Hilbert spaces: Fréchet–Riesz representation theorem, adjoint operators, self-adjoint operators, positive operators, isometric and unitary operators, normal operators.

Grading: Homework 30%, midterm 30%, final exam 40%.

Reference material (no purchase necessary):

- H. Brézis, *Functional Analysis, Sobolev Spaces, and Partial Differential Equations*. Springer, New York, 2011.
- J. Dieudonné, *Foundations of Modern Analysis*. Academic Press, New York, 1969.
- E. Kreyszig, *Introductory Functional Analysis with Applications*. Wiley, New York, 1989.
- W. Rudin, *Functional Analysis*, 2nd. ed. McGraw-Hill, New York, 1991.
- M. Schechter, *Principles of Functional Analysis*, 2nd edition. American Mathematical Society, 2001.