Course title: *MA542 – Convex Optimization Methods in Data Science*

Instructor: Patrick L. Combettes, SAS-3276, 919-515-2671, plc@math.ncsu.edu

Term: Spring 2024

Time: Tuesdays and Thursdays, from 10:15 to 11:30

Office hours: Tuesdays and Thursdays from 11:35 to 12:35 (or anytime by appointment)

Course description: This course is intended to provide an account of convex optimization methods and their applications in various areas of data science (signal and image processing, inverse problems, statistical data analysis, machine learning, classification, neural networks, etc.). The basic theory will be provided and a strong emphasis will be placed on algorithm design and concrete applications.

Student learning outcomes:

- Master the basic tools of convex analysis
- Ability to characterize solutions to convex optimization problems
- Ability to formulate standard data science problems as convex optimization problems
- Understand the structure and implementation of the main classes of algorithms for solving optimization problems arising in data science

Prerequisite: Calculus, basic linear algebra.

Detailed content:

• Overview and motivations • Iteration principles • Fixed point algorithms • Convex sets and convex cones • Best approximation paradigms • Projection methods in convex feasibility problems – applications to data fusion and image recovery • Convex functions • Conjugation of convex functions • Duality in convex optimization • Subdifferential calculus • Subgradient algorithms for convex feasibility and best approximation – applications in inverse problems • Proximity operators • Proximal calculus • Forward-backward splitting and variants (Dykstra-like methods, Chambolle-Pock algorithm, dual ascent method, etc.) • Douglas-Rachford splitting and variants (parallel proximal algorithm, alternating direction method of multipliers, composite primal-dual method, etc.) • The monotone+skew decomposition principle – primal-dual algorithms • Proximal modeling of statistical information • Proximal information extraction • Proximal sparsity enforcement • Proximal data classification • Proximal principal component analysis • Proximal image reconstruction • Proximal learning • Proximal methods for matrix-based learning • Scalability: proximal methods in big data problems • Special topics

Schedule:

- Iterative fixed point methods (3/2 weeks)
- Convex sets and projection methods (3/2 weeks)
- Convex functions and duality (3/2 weeks)
- Proximal calculus and modeling (3/2 weeks)
- Foundations of convex optimization (2 weeks)

- Proximal splitting algorithms (3 weeks)
- Specific data science applications (3 weeks)

Grading: Homework 30%, midterm exam 30%, final exam 40%.

Reference material (no purchase necessary):

- H. H. Bauschke and P. L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces, 2nd ed.* Springer, New York, 2017. Corrected reprint 2019.
- P. L. Combettes, The convex feasibility problem in image recovery, in: *Advances in Imaging and Electron Physics* (P. Hawkes, Ed.), vol. 95, pp. 155–270. Academic Press, New York, 1996.
- P. L. Combettes and J.-C. Pesquet, Proximal splitting methods in signal processing, in *Fixed-Point Algorithms for Inverse Problems in Science and Engineering,* (H. H. Bauschke et al., eds), pp. 185–212. Springer, New York, 2011.
- P. L. Combettes and J.-C. Pesquet, Fixed point strategies in data science, *IEEE Transactions on Signal Processing*, vol. 69, pp. 3878–3905, 2021.
- R. Glowinski, S. J. Osher, and W. Yin (Eds.), *Splitting Methods in Communication, Imaging, Science, and Engineering*. Springer, New York, 2016.
- S. Sra, S. Nowozin, and S. J. Wright, *Optimization for Machine Learning.* MIT Press, Cambridge, MA, 2012.