Course title: *MA716 – Advanced Functional Analysis*

Instructor: Patrick L. Combettes, plc@math.ncsu.edu

Term: Fall 2023

Time: Tuesdays and Thursdays, 15:00–16:15, SAS 1218

Office : SAS 3276

Office hours: Tuesdays and Thursdays, from 16:20 to 17:20 (or by appointment)

Course objectives: This course provides an account of essential tools in modern functional analysis in view of its applications to approximation theory, optimization, the calculus of variations, partial differential equations, game theory, inverse problems, machine learning, statistics, and control. Duality and nonlinear analysis techniques are emphasized.

Prerequisite: MA426 or equivalent.

Content:

- Vector space: algebraic duality, sets and functions in vector spaces, separation, sets associated with functions and functions associated with sets, convexity preserving transformations.
- Topology: order, nets, topological spaces, metrizability, sets and functions in topological spaces.
- Topological vector spaces: construction, locally convex spaces, duality, topology induced by a family of seminorms, separation, weak and weak$*$ topologies, inductive limits, sets and functions in topological vector spaces.
- Convex analysis in locally convex spaces: convex sets and functions, infimal convolution and post-composition, the Legendre transform, conjugate calculus, subdifferential calculus, differential calculus, optimization and duality.
- Theory of distributions: construction and basic operations, topological considerations, convolution, differentiation, Fourier transform.
- Reflexive Banach spaces: characterizations, composite minimization problems, fixed point problems, monotone operators and applications.

Grading: Homework 30%, midterm exam 30%, take-home final exam 40%.

Reference material (no purchase necessary):

- H. H. Bauschke and P. L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces,* 2nd ed. Springer, New York, 2017.
- T. Rockafellar, *Conjugate Duality and Optimization.* SIAM, Philadelphia, PA, 1974.
- L. Schwartz, *Théorie des Distributions,* 2nd ed. Hermann, Paris, 1966.
- R. E. Showalter, *Monotone Operators in Banach Space and Nonlinear Partial Differential Equations*. Amer. Math. Soc., Providence, RI, 1997.
- F. Trèves, *Topological Vector Spaces, Distributions, and Kernels.* Academic, New York, 1967.
- C. Zălinescu, *Convex Analysis in General Vector Spaces.* World Scientific Publishing, River Edge, NJ, 2002.