

Course title: MA797 – Convex Analysis

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Course objectives: Convex analysis constitutes one of the main pillars of nonlinear analysis and it is an indispensable tool for pure and applied mathematicians. Convex analysis has applications in various fields of mathematics, decision sciences, and engineering including: calculus of variations, optimization, machine learning, control theory, statistics, information theory, signal and image processing, operations research, nonlinear PDEs, probability theory, transportation theory, mechanics, mean field games, etc. The objective of the course is to provide the main bases and techniques of convex analysis, discuss convex optimization algorithms, and explore concrete applications.

Prerequisite: Calculus, basic linear algebra

Content: • Convex sets • Support functions • Generalized interiors and polarity • Convex functions • Qualification conditions • Convexity and nonexpansiveness • Infimal convolution • Infimal post composition • The Legendre transform • Subdifferential calculus • Differential calculus for convex functions • Optimality conditions • Optimality conditions • Smooth and nonsmooth optimization algorithms • Primal-dual optimization algorithms • Parametric duality • Fenchel-Rockafellar duality • Special topics: inequalities in information theory and probability, variational methods in machine learning, PDEs, signal and image processing, convex modeling in economics and statistics, etc.

Reference material (no purchase necessary):

- H. H. Bauschke and P. L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, 2nd ed. Springer, New York, 2017.
- J.-B. Hiriart-Urruty and C. Lemaréchal, *Fundamentals of Convex Analysis*. Springer, New York, 2001.
- R. T. Rockafellar, *Convex Analysis*. Princeton University Press, Princeton, NJ, 1970.